MATHS REVISION LEARNING LOG 1

Date: 11/01/2020 **Time:** 10am – 5pm

Summary

Spent the first few hours on Computation with Integers (Positive Integers Only). For the remainder of the session, we reviewed content from the Mathematics Preliminary Course (notes taken from Maths in Focus).

Contents

Computation with Integers (Positive Integers Only) – Stage 4

Questions & Points that came up:

- Computation with integers breakdown:
  - Four operations (addition, subtraction, multiplication, division)
  - Grouping symbols (brackets, parentheses, braces)
  - Order of operations (BODMAS/BEDMAS/PEDMAS)
  - Arithmetic laws (associativity, commutativity, distributivity)
  - Process of applying operations to multi-digit numbers (carrying/borrowing 1)
- Is memorising timetables important for students?
  - On one hand, it'd make mental arithmetic for most topics slightly faster
  - It might be more useful to teach about primes (particularly 2, 3, 5, 7 and maybe 11) and their role in multiplication
  - See: Prime Climb https://primeclimbgame.com/
- Long division – why do we start long division from the largest place value and move towards the smallest?
  - In addition, subtraction and multiplication, we begin by applying the operation to the smallest place value (usually the units/ones), then move left to higher place values (e.g. tens, hundreds)
  - However, for division we seem to start from the left and move right
  - Two explanations:
    - http://mathcentral.uregina.ca/QQ/database/QQ.09.99/dave1.html (argument about efficiency)
    - https://www.quora.com/Why-do-we-always-start-division-from-the-left-hand-side (argument about significant digits)
- Long Division – what's an easy way of explaining the long division algorithm https://en.wikipedia.org/wiki/Long_division
  - The process of long division is not isolated to division; it involves multiplication and subtraction as well
  - There are some kinaesthetic models for long division available online; these may be helpful but don't seem 100% satisfying as explanations
  - It seems like most people (myself included) were taught the algorithm process (whether through rote learning or by song), but not why it works (which is of interest)
  - This one still an open question/line of inquiry
- Place values – being able to convert from larger to smaller place values (e.g. 1 hundred = 10 tens) seems crucial for the division process (same thing in reverse for the other operations)
  - It's important to ensure students entering secondary education recognise this number relationship; could be an opportunity to revisit the base 10 blocks

Basic Arithmetic – Year 11

Note: These topics were taken from outdated textbooks that have not been updated for the new Stage 6 NSW Mathematics syllabus.

Observations/things you may have forgotten:

- Irrationality proofs are now in the Extension 2 syllabus (e.g. prove that $\sqrt{2}$ is irrational)
- Modern calculators typically have a memory function to store values (i.e. you can assign numerical values to certain letters); these can make longer calculations a bit easier to type into the calculator (sometimes the calculator starts to lag if there are too many digits/operations going on)
- Fix Key can be used to automatically round numbers to certain decimal places
- To explain why negative times (or divide) negative results in a positive answer, use patterns for demonstration (e.g. first demonstrate 3 x 2, 3 x 1, 3 x 0, 3 x -1 ... then demonstrate the same pattern for -3, going from -3 x 3 to -3 x -1 and observing the trend)
- Tricky index laws:

$$x^{-n} = \frac{1}{x^n} \qquad a^{\frac{1}{n}} = \sqrt[n]{a} \qquad a^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{a}} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

- Significant figures
  - Count significant figures from the first non-zero digit (from the left to right)
  - Non-zero digits are always significant; zeroes are not significant except between two non-zero digits or at the end of a decimal number
  - We can rewrite numbers rounded off to a number of significant figures using scientific notation (e.g. 14300 = 1.43×10^4 (3 s.f.)
- Changing Log Base: $\log_a b = \frac{\log_{10} b}{\log_{10} a}$
- Absolute value of x is defined as the following:
  - $|x| = \begin{cases} x \; when \; x \geq 0 \\ -x \; when \; x < 0 \end{cases}$
  - We can better understand this using an example: |3| would equal 3 since $3 \geq 0$, but $|-3|$ would equal –(-3)=3 since $-3 < 0$.
  - This will have implications for how we solve absolute value questions later on
- Absolute Value Property: $|a - b| = |b - a|$
  - To expand on this: $|a - b| = |-(b - a)| = |-1| \cdot |b - a| = 1 \cdot |b - a| = |b - a|$
- Triangle Inequality: $|a + b| \leq |a| + |b|$
  - This is simple to verify by substituting values for a and b (when a and b are both non-negative, it is an equality)
  - You can also interpret this property using geometry: https://en.wikipedia.org/wiki/Triangle_inequality (see the first picture)

Algebra and Surds – Year 11

Observations/things you may have forgotten:

- Binomial – mathematical expression consisting of two terms (e.g. $x + 3$)
- There's three (possibly more) methods for factorising trinomials (i.e. expressions with three terms, e.g. $x^2 - 4x + 3$)
  - Guess & Check; Cross Method; PSF Method (Product of first and last terms, sum or middle term, factors of P that give S)
  - Personally, only use Guess & Check these days
- Sums and differences of 2 cube
  - $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
  - $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- Completing the square (on $a^2 + pa$): divide p by 2 and square it
  - $a^2 + pa + \left(\frac{p}{2}\right)^2 = \left(a + \frac{p}{2}\right)^2$

Equations – Year 11

Observations/things you may have forgotten:

- Absolute value and inequalities
  - I think the visual interpretation (using the number line) of absolute value and inequalities makes solving these kinds of questions easier
  - First: absolute value of a number denotes magnitude, or the distance from zero (either direction) on a number line
  - So, e.g. $|x| < 2$ are all the numbers that are within 2 units distance from 0 (e.g. 1.5, -1, 0)
  - E.g. $|x| \geq 2$ are all the numbers that are equal or greater than 2 units distance from 0 (e.g. 3, -4, 5)
  - We can simplify inequalities like $|x| < 2$ as $-2 < x < 2$ (because of the above interpretation, mathematics works out fine here); however $|x| > 2$ requires breaking it down into two inequalities ($x > 2$ and $x < -2$)
  - It's strongly recommended after solving an inequality to re-substitute it back in to see if it holds (some solutions may not hold after re-substitution)
- Simultaneous equations with 3 unknown variables
  - You will typically be given 3 equations, each with 3 unknown variables (e.g. a, b, c)
  - Add two equations together (to cancel out one of the variables), then do the same with another pair of equations
    - From this you should obtain two new equations, each with 2 unknown variables (e.g. a and b); add the two equations together to cancel out one of the variables and obtain the value for the other variable
    - Substitute this value into one of the two equations you produced (i.e. the ones with 2 variables) to obtain the value of the second variable, then substitute the value of the two variables back into one of the three original equations

Geometry – Year 11

Observations/things you may have forgotten:

- Lines are infinite; intervals (parts of line) have endpoints
- Median (triangle) – line from a corner to the midpoint of the opposite side in a triangle (i.e. the line bisects the opposite line)
- Altitude (triangle) – perpendicular line from a corner to the opposite side of a triangle (also known as the height of the triangle)
- Congruency Tests (same shape AND size)
    - SSS – all 3 pairs of corresponding sides EQUAL
    - SAS – 2 pairs of corresponding sides and their included angle (angle between the two sides) are EQUAL
    - AAS – 2 pairs of angles, 1 pair of corresponding sides equal
    - RHS – both have right angles, hypotenuses equal, 1 other pair of corresponding sides equal
- Similarity Tests (same shape, different sizes)
    - Equiangular – three pairs of corresponding angles equal
        - Note: if 2 pairs are equal, the 3rd pair has to also be equal
    - Three pairs of corresponding sides are in proportion
    - Two pairs of sides in proportion, included angles equal
- Ratio of intercepts (from similar triangles) – also known as Thales' theorem/basic proportionality
    - https://en.wikipedia.org/wiki/Intercept_theorem can read more here
    - When two (or more) transversals cut a series of parallel lines, the ratios of their intercepts are equal, i.e. $AB{:}BC = DE{:}EF$ or $\frac{AB}{BC} = \frac{DE}{EF}$
    - The most essential bit is being able to recognise intercepts: they are the line segments produced when a line intersects with the parallel lines
    - Sometimes, the lines will intersect at a point (e.g. in overlapping triangles) – you can still find intercepts (labelling will differ though)

Functions and Graphs – Year 11

Observations/things you may have forgotten:

- Odd functions: for all x, $f(x) = f(-x)$
    - A consequence of this is that even functions are symmetrical along the y-axis
- Even functions: for all x, $f(-x) = -f(x)$
    - A consequence of this is that odd functions have point symmetry about the origin
    - If the graph doesn't pass through the origin – won't be odd

[STOPPED HERE]

Other Ideas

- Getting students to create their own probability devices (e.g. custom dice, spinners) is easy, fun and rich in possibilities for mathematical/ethical discussions
- Are word problems the way to go for first introducing division (e.g. sharing scenarios)?

Future Plans

- Will continue looking through Year 11 content:
    - Functions and Graphs
    - Trigonometry
    - Linear Functions
    - Introduction to Calculus
    - The Quadratic Function
    - Locus and the Parabola (will likely ignore this)